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**STATUS OF TRIMAXIMAL NEUTRINO MIXING**

W. G. Scott

*Rutherford-Appleton Laboratory, Chilton, Didcot, Oxon.**OX11-0QX, UK**E-mail: w.g.scott@rl.ac.uk*

Trimaximal mixing is the mixing hypothesis with maximal symmetry. In trimaximal mixing there remains a still worrying conflict between the large values of  $\Delta m^2$  preferred by the atmospheric fits (possibly supported by the early K2K data) and reactor limits on  $\nu_e$ -mixing. However, the latest solar results do seem to point to energy-independent (ie. ‘no-scale’) solar solutions, like the trimaximal solution.

**1 Introduction**

At the last IDM meeting (IDM98) I reviewed [1] neutrino oscillations, mentioning trimaximal mixing [2,3] of course, but giving substantive emphasis to the so-called bimaximal scheme [4] which was new at that time. This year, I sense that bimaximal mixing is in no particular need of any ‘hard-sell’ from me, and I plan therefore to concentrate on trimaximal mixing, which arguably merits a little more ‘air-time’ than it sometimes gets at this point.

After all, bimaximal mixing (in its original form) may be seen as just the minimal deformation [5] of trimaximal mixing obtained enforcing a zero in the top right-hand ( $e3$ ) corner of the trimaximal mixing matrix to account for the latest reactor data. Evidently (Eq. 1) symmetry between all three generations

$$\begin{array}{ccc}
 \begin{array}{c} \text{trimaximal mixing} \\ \nu_1 \quad \nu_2 \quad \nu_3 \\ e \left( \begin{array}{ccc} 1/3 & 1/3 & 1/3 \\ \mu \left( \begin{array}{ccc} 1/3 & 1/3 & 1/3 \\ \tau \left( \begin{array}{ccc} 1/3 & 1/3 & 1/3 \end{array} \right) \end{array} \right) \end{array} \right) \\ \end{array} & \longleftrightarrow & \begin{array}{c} \text{bimaximal mixing} \\ \nu_1 \quad \nu_2 \quad \nu_3 \\ e \left( \begin{array}{ccc} 1/2 & 1/2 & . \\ \mu \left( \begin{array}{ccc} 1/4 & 1/4 & 1/2 \\ \tau \left( \begin{array}{ccc} 1/4 & 1/4 & 1/2 \end{array} \right) \end{array} \right) \end{array} \right) \\ \end{array}
 \end{array} \quad (1)$$

is sacrificed in the bimaximal scheme, but clearly  $\nu_1 \leftrightarrow \nu_2$  as well as  $\mu \leftrightarrow \tau$  symmetry do survive [5] (note Eq. 1 gives the  $|U_{l\nu}|^2$ ). It should be mentioned that the famous Fritzsch-Xing hypothesis [6] did in fact predict  $U_{e3} = 0$ , but has otherwise less symmetry than either the trimaximal or bimaximal schemes. Altarelli and Feruglio [7] usefully generalised the bimaximal scheme, retaining the last column of the original bimaximal form (Eq. 1 - RHS), but parametris-ing the first two columns in terms of a general mixing angle  $\theta$ , to be determined.

In praise of trimaximal mixing, the trimaximal mixing matrix (Eq. 1 - LHS) is clearly especially symmetric, extremal/optimal and arguably ‘natural’

from a number of points of view. The importance of  $Z_3$  symmetry seems to be generally recognised [8]. In analogy to the Uncertainty Principle the lepton flavour information is uniformly spread over the neutrino mass spectrum and vice versa (cf. also, the famous ‘hexacode’ over  $F_4$ ). For trimaximal mixing the Jarlskog parameter  $J_{CP}$  [9] takes its extremal value  $J_{CP} = 1/(6\sqrt{3})$  such that (vacuum) CP and T violating asymmetries are maximised.

## 2 Data at the Atmospheric Scale

The atmospheric neutrino scale defined by fits to the SUPER-KAMIOKANDE data (Fig. 1) is currently  $\Delta m^2 \simeq 3 \cdot 10^{-3} \text{ eV}^2$  [10]. Upcoming  $\mu$ -events (Fig. 1a)

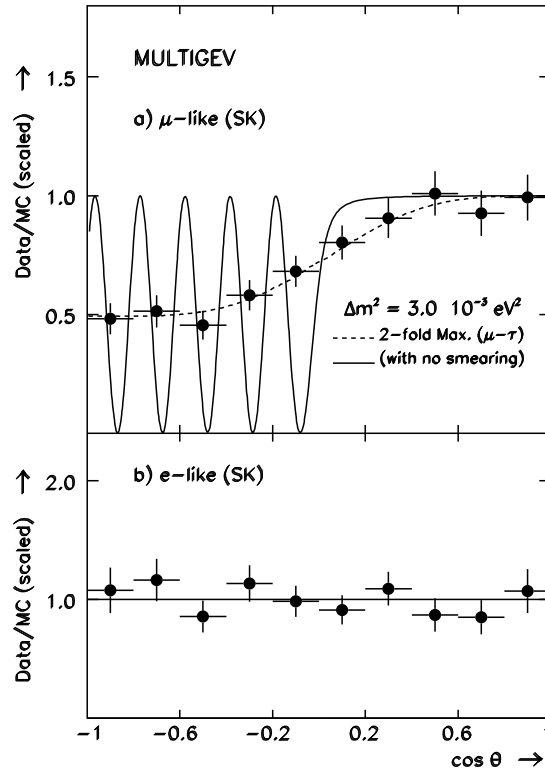


Figure 1: The multi-GeV zenith angle distributions for a)  $\mu$ -like and b)  $e$ -like events in SUPER-K. The solid curve is the full oscillation curve for bimaximal mixing (Eq. 1 - RHS) with  $\Delta m^2 = 3 \cdot 10^{-3} \text{ eV}^2$  for a representative neutrino energy  $E = 3 \text{ GeV}$ . The dashed curve shows the effect of angular smearing and averaging over neutrino energies.

are seen to be suppressed by a factor  $P(\mu \rightarrow \mu) \simeq 1/2$ . Solving the equation  $(1-x)^2 + x^2 = 1/2$  yields  $x \equiv |U_{\mu 3}|^2 \simeq 1/2$  as in Eq. 1 (RHS). No deviation is seen for  $e$ -events (Fig 1b), but  $\phi(\nu_\mu)/\phi(\nu_e) \simeq 2/1$ , coupled with matter effects (especially if  $\Delta m^2 \lesssim 3 \cdot 10^{-3} \text{ eV}^2$ , see below) gives low sensitivity to  $\nu_e$ -mixing.

Reactor experiments on the other hand, specifically CHOOZ [11] and PALO-VERDE[12], *do* rule out large  $\nu_e$  mixing over (almost) all of the  $\Delta m^2$ -range currently favoured in the atmospheric neutrino experiments. While the atmospheric experiments claim  $10^{-3} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-2} \text{ eV}^2$ , at around the 99% confidence level, the reactor experiments require  $\Delta m^2 \lesssim 10^{-3} \text{ eV}^2$  unless  $U_{e3}$  is small:  $|U_{e3}|^2 \lesssim 0.03$ . The near non-overlap of these two different  $\Delta m^2$  ranges underlies the current popularity of the (generalised) bimaximal scheme(s) discussed above. Of course trimaximal mixing predicts  $|U_{e3}|^2 = 1/3$

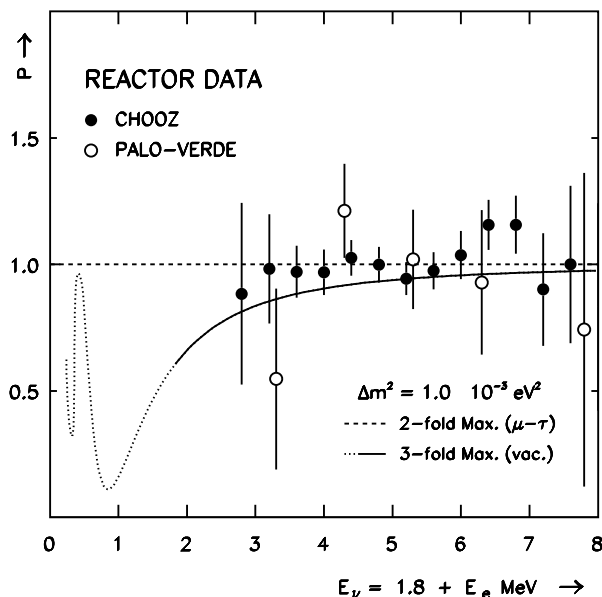


Figure 2: The survival probability  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  measured in the CHOOZ and PALO-VERDE reactor experiments (filled and open data points) compared to the trimaximal mixing prediction for  $\Delta m^2 = 1.0 \cdot 10^{-3} \text{ eV}^2$  (solid curve).

which is large, so if trimaximal mixing is right  $\Delta m^2$  is at least well determined, ie.  $\Delta m^2 \simeq 10^{-3} \text{ eV}^2$  [3] with rather little margin for error. Clearly (Fig. 2) trimaximal mixing with  $\Delta m^2 = 1.0 \cdot 10^{-3} \text{ eV}^2$ , would fit the combined CHOOZ and PALO-VERDE data very nicely, given only a modest re-scaling of the CHOOZ data by  $\sim -8\%$  (the quoted error on the CHOOZ flux is  $\sim \pm 3\%$ ).

Long-baseline accelerator experiments like K2K [13] have well defined  $L/E$ , so there is no way that  $\Delta m^2$  is overestimated by uncertain angular smearing effects as perhaps in the atmospheric experiments. Worryingly for trimaximal mixing, K2K have 27 FC events in fiducial volume with 26.6 events expected for  $\Delta m^2 \simeq 3 \cdot 10^{-3} \text{ eV}^2$ , with very much closer to 40.3 events (the no-oscillation

Conference	Evts. Obs. FC IFV	Evts. Expd. (no osc.)
QUEBEC	3	12.3
NU2000	14	16.9
ICHEP2000	10	11.1
Total	27	40.3

Table 1: K2K fully contained events in fiducial volume, seen vs. expected (for no oscillation) (chronologically ordered independent samples conveniently separated here by conference.)

expectation) expected for  $\Delta m^2 \simeq 1.0 \cdot 10^{-3} \text{ eV}^2$ . As the K2K experimenters have themselves pointed out however [13], there is something slightly ‘odd’ about the distribution of events versus chronological expectation (Table 1), with most of the deficit apparently coming from the 1999 running.

### 3 The Solar Data

The latest SUPER-K data on the solar suppression [14] extend the electron recoil spectrum down to  $E > 5 \text{ MeV}$  and start to be convincingly ‘consistent with flat’ (ie. with energy independence). Assuming BP98 fluxes [15], the overall suppression  $S \simeq 0.47$ . Correcting for the neutral current contribution (Fig. 3) we find  $P(e \rightarrow e) \simeq 0.38$ , not so very different from the HOMESTAKE result  $P(e \rightarrow e) \simeq 0.33$ . The solid curve in Fig. 3 is the postdiction of the ‘optimised’

$$\begin{array}{c} \text{‘optimised’} \quad \text{bimaximal} \quad \text{mixing} \\ \nu_1 \quad \nu_2 \quad \nu_3 \\ (|U_{\ell\nu}|^2) = \begin{array}{c} e \\ \mu \\ \tau \end{array} \begin{pmatrix} 2/3 & 1/3 & . \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} \end{array} \quad (2)$$

bimaximal hypothesis (Eq. 2) with  $\Delta m'^2 = 5.6 \cdot 10^{-5} \text{ eV}^2$ . The ‘optimised’ bimaximal form is readily obtained from the ‘generalised’ bimaximal scheme [7] by setting  $\sin^2 \theta = 1/3$ , and we proposed it [3,16] only as the best ‘straw-man’ rival to trimaximal mixing, with the possibility to exploit the LA-MSW solution (Fig. 3). Of course, energy *in*-dependent solar solutions, like the trimaximal mixing solution (Fig. 4), remain a priori much more plausible [16]. It is interesting to see that the early SNO data [17] support energy independence.

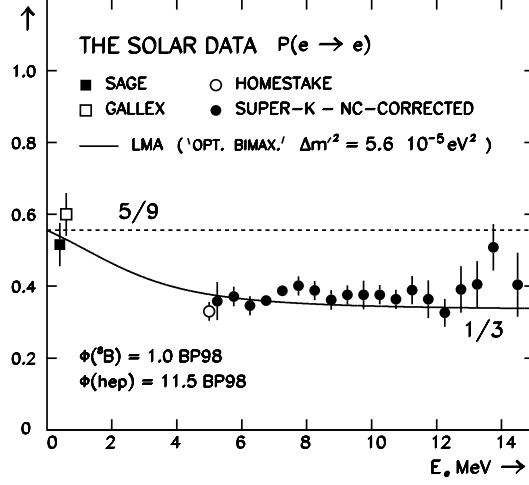


Figure 3. The SUPER-K solar data [14] after the NC subtraction vs. recoil energy  $E_e$ , assuming the BP98  $^8B$ -flux [5] (with rescaled hep). SAGE, GALLEX and HOMESTAKE points also shown but versus  $E_\nu$ . The solid curve is 'optimised' bimaximal mixing (Eq. 2) with  $\Delta m'^2 = 5.6 \times 10^{-5} \text{ eV}^2$ , giving  $P(e \rightarrow e) = 1/3$  in the 'bathtub' (and  $5/9$  otherwise).

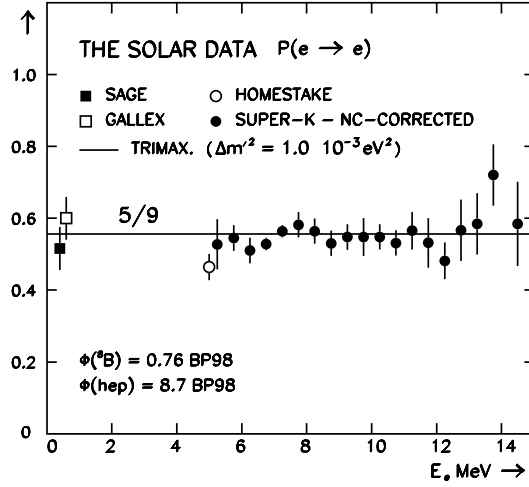


Figure 4. As for Fig. 3, except for an arbitrary rescaling of BP98  $^8B$ -flux by  $-24\%$ . The line is the trimaximal mixing prediction  $P(e \rightarrow e) = 5/9$  independent of energy. Note that Eq. 2 (with or without a  $\nu_1 \leftrightarrow \nu_2$  column interchange if desired) likewise gives  $P(e \rightarrow e) = 5/9$  independent of energy outside the 'bathtub' region. Thus Eq. 2 can never be excluded based on the solar data alone, underlining again the importance of KAMLAND [12], K2K [13] etc.

In SUPER-K, a previous  $2\sigma$  day/night asymmetry  $A = 0.065 \pm 0.031 \pm 0.013$  has fallen with increased statistics to  $1\sigma$  significance:  $A = 0.034 \pm 0.022 \pm 0.013$  [14]. A day/night effect would have been the ‘smoking-gun’ of the MSW or VO solutions. Instead, essentially *all*  $10^{-10} \text{ eV}^2 \lesssim \Delta m'^2 \lesssim 10^{-3} \text{ eV}^2$  are now allowed, with (near-)maximal mixing, *except* those explicitly excluded (eg.  $2 \cdot 10^{-7} \text{ eV}^2 \lesssim \Delta m'^2 \lesssim 2 \cdot 10^{-5} \text{ eV}^2$ ) by the absence of a day/night effect.

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